Mathematics Standards Rationale

The four National Council of Teachers of Mathematics (NCTM) standards of problem solving, reasoning, communication and connections are goals interwoven throughout the Arizona mathematics standards. These goals are the reason people study and use mathematics, and they should permeate everything we do in and outside the classroom.

Whenever possible, mathematical learning should be placed in a broader, problem solving context and evaluated through performance assessments. In this setting, students discover questions involving numbers or equations from a real-world context which lead to answers that have meaning. Ultimately, all problems should be application problems; more ideally, problems should be presented in the broader context of an investigation or project. This way the students use problem solving, reasoning, communication and connections in every mathematical activity. The spirit of these four goals is a mathematical apprenticeship in which the students solve problems on a daily basis, much as mathematics is used in the real world.

Even the youngest students can use mathematics to solve social science problems, engineering problems and business problems in a meaningful way. As early as possible, students should learn that mathematics is everywhere in the world around them. They should realize that in the real world not all answers are small whole numbers; instead they can be large or small and/or have a fractional part.

As students develop their ability to perceive and conceptualize in problem solving, they should reason about the mathematics they do. Teachers should guide students to ask such questions as: *Does the answer make sense? Are there other ways to arrive at the answer? Does the answer bring up more questions? Can I answer those? What other information would I need?* It is this kind of reasoning that enriches a mathematical educational experience. If students do answer such significant questions, they then naturally apply mathematics in everyday life. Without this guidance, they remain mathematically deficient.

Teachers should engage students in mathematical discourse at all stages of learning. Mathematics was developed as a means to communicate about quantities, logical relationships and unknowns. To use this language, students should communicate (both orally and in writing) everything they do mathematically. They should explain their mathematical thinking through language, through models, graphically, geometrically, numerically and algebraically. Students should be encouraged to express themselves in as many ways as possible and to learn to translate between one mathematical language and another.

Students should regularly see the mathematical connections within the course of an investigation or project. They must experience mathematics primarily through its connections to other disciplines. For too long we have structured our curricula to reach the few who will use mathematics in isolation rather than the majority who will apply it to their work or study in other fields.

A variety of tools should be available to students as they develop concepts and understandings of mathematics. Graphing calculators and computers should be standard equipment in mathematics classrooms. New technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

As the four essential standards–problem solving, reasoning, communication and connections– and the implementation of technology become functioning parts of our curricula, we can expect all Arizona students will develop the mathematical power to confidently handle the future. They will be able to face the world knowing that they can not only merely compute but also that they can use meaningful mathematics to solve real problems.

The organization of the content in these standards is designed for readability purposes and is not intended to dictate sequence or to define the structure of courses. Topics from all six mathematics standards need to be continuously integrated within the curricula.

Table 1. Mathematics Standards

STANDARD 1: **Number Sense**

Students develop number sense and use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to determine the reasonableness of results.

STANDARD 2: **Data Analysis and Probability**

Students use data collection and analysis, statistics, and probability to make valid inferences, decisions and arguments and to solve a variety of real-world problems.

STANDARD 3: **Patterns, Algebra and Functions**

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem solving situations.

STANDARD 4: **Geometry**

Students use geometric methods, properties and relationships as a means to recognize, draw, describe, connect and analyze shapes and representations in the physical world.

STANDARD 5: **Measurement and Discrete Mathematics**

Students make and use direct and indirect measurement, metric and U.S. customary, to describe and compare the real world and to prepare for the study of discrete functions, fractals and chaos which have evolved out of the age of technology.

STANDARD 6: **Mathematical Structure/Logic**

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

MATHEMATICS STANDARDS
BY LEVEL: ESSENTIALS (4-8)

STANDARD 1: NUMBER SENSE

Students develop number sense and use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to determine the reasonableness of results.

Note: Certain performance objectives may appear to be misnumbered. This document has been reformatted and the numbering from the original document has been retained.

- **1M-E1. Read, write and order integers, whole numbers and rational numbers**

  (Grades 4-5)

  PO 1. Compare and order using concrete or illustrated models
  - A. whole numbers (to millions)
  - B. common fractions (halves, thirds, fourths, eighths)
  - C. decimals (thousandths)

  PO 2. Represent place value using concrete or illustrated models
  - A. whole numbers (millions), decimals (thousandths)

  PO 3. Read and write whole numbers, integers, common fractions and decimals using real-world situations
  - A. whole number (millions), decimals (thousandths), fractions (halves, thirds, fourths, eighths)

  (Grade 6-8)

  PO 1. Compare and order using concrete or illustrated models
  - D. rational numbers (e.g., -5, 1.2, 1 3/4, square root of 16)

  PO 2. Represent place value using concrete or illustrated models
  - B. rational numbers (millions to millionths)

  PO 3. Read and write whole numbers, integers, common fractions and decimals using real-world situations
  - B. rational numbers (millions to millionths)

- **1M-E2. Relate the basic arithmetic operations to one another (e.g., multiplication and division are inverse operations)**

  (Grades 4-5)

  PO 1. Represent the process of multiplication as repeated addition, using **concrete or illustrative models**
  - A. whole numbers

  PO 2. Represent the process of division as repeated subtraction, partitioning a group and partitioning a whole, using **concrete or illustrative models**
  - A. whole numbers

PO 3. Write the family of equations using inverse operations for a given set of numbers
   A. whole numbers with addition/subtraction [(4 + 5 = 9, 5 + 4 = 9, 9 – 4 = 5, 9 – 5 = 4) and multiplication/division]

(Grades 6-8)

PO 1. Represent the process of multiplication as repeated addition, using **concrete or illustrative models**
   B. fractions and decimals

PO 2. Represent the process of division as repeated subtraction, partitioning a group and partitioning a whole, using **concrete or illustrative models**
   B. fractions and decimals

PO 3. Write the family of equations using inverse operations for a given set of numbers
   B. positive fractions and decimals, integers with addition/subtraction and multiplication/division

- **1M-E3. Demonstrate proficiency with the operations of multiplication and division of whole numbers**

(Grades 4-5)

PO 1. Calculate multiplication/division
   A. three-digit by two-digit to find the product
   B. facts through 12
   C. mental math and estimation with multiples of 10
   D. one-digit divisor to find quotient with remainder

PO 2. Calculate multiplication and division problems using contextual situations

(Grades 6-8)

PO 1. Calculate multiplication/division
   E. two-digit divisor, with remainders and rounding in context (e.g., percentages and money)

PO 2. Calculate multiplication and division problems using contextual situations

- **1M-E4. Develop and apply number theory concepts (e.g., primes, factors and multiples) to represent numbers in various ways**

(Grades 4-5)

PO 1. State the factors for a given whole number

PO 4. Sort numbers by their properties
   A. odd, even

(Grades 6-8)

PO 2. Factor a whole number into a product of its primes (prime factorization)

PO 3. Identify greatest common factor and least common multiples for a set of whole numbers

PO 4. Sort numbers by their properties

B. prime, composite, square, square root

PO 5. Simplify numerical expressions using order of operations

- **1M-E5. Represent and use numbers in equivalent forms (integers, fractions, percent, decimals, exponents, scientific notation and square roots)**

(Grades 4-5)

PO 2. Demonstrate the relationship and equivalency among

A. decimals, fractions and percents (e.g.,1/2 = .5 = 50% with halves, fourths and tenths)

(Grades 6-8)

PO 1. Add, subtract, multiply and divide integers, positive fractions and decimals

PO 2. Demonstrate the relationship and equivalency among

B. decimals, fractions, ratios, percents

PO 3. Factor numbers into prime form and express in exponential form

PO 4. Convert standard notation to scientific notation and vice versa with positive exponents

PO 5. Determine the square root of a perfect square

- **1M-E6. Recognize that the degree of precision needed in calculating a number depends on how the results will be used and the instruments used to generate the measurements**

(Grades 4-5)

PO 2. Apply the appropriate strategy (e.g., estimation, approximation, rounding or exact numbers) when calculating to solve problems

PO 3. Demonstrate/describe the magnitude of

A. whole numbers (e.g., "How many apples in the orchard?")

Note: We recommend that the following be assessed at the district level:

PO 4. Interpret calculations and calculator results within a contextual situation

(Grades 6-8)

PO 1. Express answers to the appropriate place or degree of precision (e.g., time, money, pi)

PO 2. Apply the appropriate strategy (e.g., estimation, approximation, rounding or exact numbers) when calculating to solve problems

PO 3. Demonstrate/describe the magnitude of

B. rational numbers (e.g., "How small is a bacterium?")

Note: We recommend that the following be assessed at the district level:

PO 4. Interpret calculations and calculator results within a contextual situation

STANDARD 2: DATA ANALYSIS AND PROBABILITY

Students use data collection and analysis, statistics, and probability to make valid inferences, decisions and arguments and to solve a variety of real-world problems.

- **2M-E1. Construct, read, analyze and interpret tables, charts, graphs and data plots (e.g., box-and-whisker, stem-and-leaf, and scatter plots)**

  (Grades 4-5)
  PO 1. Construct
  A. bar graphs, line graphs, frequency tables and Venn diagrams
  PO 2. Interpret and analyze data from graphical representations and draw simple conclusions
  A. bar graphs, line graphs, circle graphs, frequency tables and Venn diagrams

  (Grades 6-8)
  PO 1. Construct
  B. histograms, stem-and-leaf plots, scatter plots, circle graphs, and flow charts
  PO 2. Interpret and analyze data from graphical representations and draw simple conclusions
  B. histograms, stem-and-leaf plots, scatter plots, circle graphs and flow charts
  PO 3. Choose an appropriate graphical format to organize and represent data

- **2M-E2. Make valid inferences, predictions and arguments based on statistical analysis**

  (Grades 4-5)
  PO 1. Formulate predictions from a given set of data and justify predictions
  PO 2. Compare a given prediction with the results of an investigation

  (Grades 6-8)
  PO 1. Formulate predictions from a given set of data and justify predictions
  PO 2. Compare a given prediction with the results of an investigation
  PO 3. Critique the conclusions and recommendations of others' statistics
  PO 4. Consider the effects of missing or incorrect information

- **2M-E3. Display and use measures of range and central tendency (i.e., mean, median and mode)**

  (Grades 4-5)
  PO 1. Find the mean, median, mode and range of data using **concrete and illustrative models**

(Grades 6-8)

PO 2. Find the mean, median, mode and range of a data set

PO 3. Choose appropriate measures of central tendencies to describe given or derived data

- **2M-E4. Use counting strategies to determine all the possible outcomes of a particular event (e.g., the number of ways students can line up to have their pictures taken)**

(Grades 4-5)

PO 1. Find all possible outcome sets involving
   A. two sets of objects (e.g., shirts and pants)

(Grades 6-8)

PO 1. Find all possible outcome sets involving
   B. two or more sets of objects

PO 2. Find all possible arrangements given a set (e.g., "How many ways can you arrange a set of books on a shelf?")

- **2M-E5. Determine probabilities through experiments and/or simulations and compare the results with the mathematical expectation**

(Grades 4-5)

PO 1. Make predictions from the results of a student-generated experiment (empirical probability)
   A. single events (e.g., spinners)

PO 3. Describe events that are certain or impossible

PO 5. Identify outcomes that are more likely, less likely or equally likely to occur

(Grades 6-8)

PO 1. Make predictions from the results of a student-generated experiment (empirical probability)
   B. two-stage events (e.g., two spinners)

PO 2. Determine and compare experimental (empirical) and mathematical (theoretical) probabilities (e.g., flipping two-colored counters)

PO 4. Express probability as a fraction, zero or one

STANDARD 3: PATTERNS, ALGEBRA AND FUNCTIONS

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem-solving situations.

- **3M-E1. Use algebraic methods (write number sentences, in the form of expressions and equations) to explore, model and describe patterns and functions involving numbers, shapes, data, graphs and data plots**

(Grades 4-5)

PO 1. Extend simple geometric and number patterns (e.g., 1, 1, 2, 1, 1, 3, 1, 1, 4 . . .)
PO 2. Create simple geometric and number patterns
PO 3. Describe a rule for a simple pattern (e.g., 5, 10, 15, 20 . . . rule = add five or count by fives)

(Grades 6-8)

PO 4. Generate patterns using algebraic expressions

- **3M-E2. Describe, represent and analyze patterns and relationships using shapes, tables, graphs, data plots, verbal rules and standard algebraic notation**

  Note: This concept is covered in 3M-E1 and 3M-E4

- **3M-E3. Describe the concepts of variables, expressions, equations and inequalities**

  (Grades 4-5)

  Note: There are no POs at this level

  (Grades 6-8)

  PO 1. Describe and use variables in a contextual situation
  PO 2. Evaluate an expression using substitution with four basic operations on whole numbers
  PO 3. Translate a written phrase to an algebraic expression and vice versa (words to symbols and symbols to words) (e.g., the quotient of x and y)
  PO 4. Express a simple inequality from a contextual situation (e.g., Joe earns more than $5.00 an hour: therefore, $x > 5$)

- **3M-E4. Analyze functional relationships to explain how a change in one variable results in a change in another**

  (Grades 4-5)

  PO 1. Describe a real-life situation in which a change in one variable results in the change of the other (e.g., temperature in the classroom goes up and the amount of clothing goes down)
  PO 3. Compute an "output" for a given "input" in a function

  (Grades 6-8)

  PO 2. Produce the rule (function) that explains the relationship (pattern) between the numbers when a change in the first variable affects the second variable (T-chart, two-row table, or input/output machine)
  PO 4. Complete a T-chart for a given rule

- **3M-E5. Use patterns and functions to represent and solve problems both formally and informally (e.g., measuring the height a ball bounces by dropping different balls from different starting heights)**

  (Grades 4-5)

  Note: There are no POs at this level

  (Grades 6-8)
  PO 1. Solve a problem given a pattern both formally and informally (e.g., "In a patterned necklace, how many red and green beads do you need for a 20-inch necklace?")

- **3M-E6. Distinguish between linear and nonlinear functions through investigations**

  (Grades 4-5)

  Note: There are no POs at this level

  (Grades 6-8)
  PO 1. Distinguish between linear and nonlinear functions, given graphic examples

- **3M-E7. Solve simple linear equations and inequalities using a variety of methods (e.g., informal, formal, graphical) and a variety of manipulatives**

  (Grades 4-5)
  PO 1. Solve equations using
  A. whole numbers with one variable--one step
  PO 3. Graph given data points to represent a linear equation
  A. on a coordinate grid with whole numbers

  (Grades 6-8)
  PO 1. Solve equations using
  B. whole numbers with one variable--multiple steps
  PO 2. Solve linear (first degree) equations using models/manipulatives, symbols and/or graphing in a one-step equation
  PO 3. Graph given data points to represent a linear equation
  B. in (x, y) form using all four quadrants of a coordinate grid

- **3M-E8. Develop, analyze and explain methods for solving proportions**

  (Grades 4-5)

  Note: There are no POs at this level

(Grades 6-8)

PO 1. Describe how to solve a problem in context using a proportion
PO 2. Compare quantities using ratios
PO 3. Solve proportions using formal (e.g., cross product) or informal methods (e.g., diagrams, geometric models)

STANDARD 4: GEOMETRY

Students use geometric methods, properties and relationships as a means to recognize, draw, describe, connect, and analyze shapes and representations in the physical world.

- **4M-E1. Visualize and draw two- and three-dimensional geometric figures with special attention to analyzing and reasoning informally about their properties (e.g., parallelism, perpendicularity and congruence)**

(Grades 4-5)

PO 1. Classify two-dimensional shapes and three-dimensional figures by their properties
  A. by sight
PO 2. Identify the properties of geometric figures using appropriate terminology and vocabulary (e.g., parallelism, perpendicularity and congruency)
  A. two-dimensional shapes (three- and four-sided polygons)
PO 3. Draw or build two-dimensional shapes by applying significant properties of each (e.g., draw a rectangle with two sets of parallel sides and four right angles)

(Grades 6-8)

PO 1. Classify two-dimensional shapes and three-dimensional figures by their properties
  B. by properties
PO 2. Identify the properties of geometric figures using appropriate terminology and vocabulary (e.g., parallelism, perpendicularity and congruency)
  B. three-dimensional figures (prisms)
PO 3. Draw or build three-dimensional figures by applying significant properties of each (e.g., draw a rectangle with two sets of parallel sides and four right angles)

- **4M-E2. Apply geometric properties and relationships such as congruence, similarity, angle measure, parallelism and perpendicularity to real-world situations**

(Grades 4-5)

PO 1. Design or draw a model (e.g., designing a playhouse, garden) that demonstrates basic geometric relationships, such as
  A. parallelism, perpendicularity, similarity
PO 2. Classify triangles by their angles and sides (e.g., equilateral, acute, isosceles . . .)
PO 5. Identify lines that are parallel and perpendicular
PO 6. Distinguish shapes that are congruent

(Grades 6-8)

PO 1. Design or draw a model (e.g., designing a playhouse, garden) that demonstrates basic geometric relationships, such as
   B. all of the above (A) and proportionality and congruency

PO 3. Label corresponding, supplementary and complementary angles

PO 4. Measure and label specified angles (e.g., alternate interior, obtuse, acute, right, corresponding . . .)

- **4M-E3. Perform elementary transformations (e.g., tessellations, flips, slides, rotations)**

(Grades 4-5)

PO 1. Demonstrate slide, flip or turn using concrete geometric figures

PO 2. Illustrate, using concrete or pictorial models
   A. slide, flip or turn (e.g., quilts)

PO 3. Draw or build a shape that
   A. has symmetry

(Grades 6-8)

PO 2. Illustrate, using concrete or pictorial models
   B. reflections, rotations and translations (e.g., tessellations)

PO 3. Draw or build a shape that
   B. has two or more lines of symmetry

- **4M-E4. Represent and solve problems relating to size, shape, area and volume using geometric models**

(Grades 4-5)

PO 1. Solve problems using **given** formulas for
   A. simple area and perimeter

PO 2. Identify a variety of shapes having the same perimeter and area

(Grades 6-8)

PO 1. Solve problems using **given** formulas for
   B. area, perimeter/circumference of various circles/polygons
   C. volume of prisms

PO 3. Draw or build a variety of shapes having the same perimeter and area

STANDARD 5: MEASUREMENT AND DISCRETE MATHEMATICS

Students make and use direct and indirect measurement, metric and U.S. customary, to describe and compare the real world and to prepare for the study of discrete functions, fractals and chaos which have evolved out of the age of technology.

- **5M-E1. Estimate, make and use measurements (U.S. customary and metric) to describe and make comparisons**

  (Grades 4-5)
  PO 1. Measure length, volume and weight in both U.S. customary and metric units
  PO 2. Convert measurement units to equivalent units **within** a given system (customary and metric) (e.g., 12 inches = 1 foot, 10 decimeters = 1 meter)
  PO 3. Estimate measurements for both U.S. customary and metric units within either system

  (Grades 6-8)
  PO 3. Estimate measurements for both U.S. customary and metric units within either system
  PO 4. Compare estimated measurements **between** U.S. customary and metric systems (e.g., a yard is about a meter***)***

- **5M-E2. Select and use appropriate units and tools to measure to the degree of accuracy required in a particular problem-solving situation**

  (Grades 4-5)
  PO 1. State the appropriate tool to measure in a particular situation (e.g., "What tool would you use to measure the top of your desk?")
  PO 2. State the appropriate unit of measurement in a particular situation (e.g., "What unit of measurement would you use to measure the top of your desk?")
  PO 3. Measure to the appropriate degree of accuracy to solve problems (e.g., measuring to the nearest sixteenth of an inch or using ounces, measuring to the nearest millimeter or using liters)

  (Grades 6-8)
  PO 3. Measure to the appropriate degree of accuracy to solve problems (e.g., measuring to the nearest sixteenth of an inch or using ounces, measuring to the nearest millimeter or using liters)

- **5M-E3. Estimate, use and describe measures of distance, perimeter, area, volume, capacity, weight, mass and angles**

  (Grades 4-5)
  PO 1. Differentiate between perimeter and area of quadrilaterals using concrete and illustrative models
  PO 2. Record estimates and measurements for
  A. distance
  C. perimeter
  E. area
  G. weight

(Grades 6-8)

PO 2. Record estimates and measurements for
- B. distance in scale drawings
- D. circumference
- E. area
- F. volume
- H. mass
- I. degrees of angles
- J. capacity

PO 3. Compare weight to mass and capacity to volume

- **5M-E4. Develop and use formulas and procedures to solve problems involving measurement**

(Grades 4-5)

PO 1. Develop a procedure or formula to calculate
- A. area and perimeter of simple polygons

PO 2. Use **given** formulas to find
- A. area and perimeter of simple polygons

(Grades 6-8)

PO 1. Develop a procedure or formula to calculate
- B. area of polygons and circles
- C. surface area of rectangular prisms
- D. volume of rectangular prisms

PO 2. Use **given** formulas to find
- B. circumference of a circle
- C. area of polygons and circles
- D. surface area of rectangular prisms
- E. volume of prisms

- **5M-E5. Describe how a change in the linear dimension of an object affects its perimeter, area and volume**

(Grades 4-5)

PO 1. Describe the change in perimeter and area when one dimension of an object is altered

(Grades 6-8)

PO 2. Describe the effect on perimeter, area and volume when one dimension of an object is altered

- **5M-E6. Use calculators and computers to perform basic recursive and iterative processes**

  (Grades 4-5)

  PO 1. Solve a problem using the iterative process
  - A. doubling (e.g., "If you get paid 1 cent the first day, 2 cents the second day, each day doubling the previous day's pay, how much would you get paid on the twentieth day?")

  PO 2. Generate the iterative sequence for the next six terms when given the first four terms (e.g., 4, 7, 10, 13, . . .)

  (Grades 6-8)

  PO 1. Solve a problem using the iterative process
  - B. designing a simple geometric pattern (e.g., design a basic quilt block; use it to generate the whole quilt)

  PO 3. Complete the iterative sequence (e.g., given these terms and assuming a constant difference 21, -, -, -, -, -, 63, -, -, -)

  PO 4. Generate subsequent terms of a recursive sequence (e.g., 3, 3, 6, 9, 15, . . .)

STANDARD 6: MATHEMATICAL STRUCTURE/LOGIC

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

- **6M-E1. Use models to explain how ratios, proportions and percents can be used to solve problems and apply reasoning processes, such as spatial reasoning and reasoning with proportions and graphs**

  (Grades 4-5)

  Note: There are no POs at this level

  (Grades 6-8)

  PO 1. Communicate how to solve problems involving ratios, proportions and percents using concrete and illustrative models

- **6M-E2. Construct, use and explain algorithmic procedures for computing and estimating with whole numbers, fractions, decimals and integers**

  (Grades 4-5)

  PO 1. Design a method with a series of defined steps for solving a problem; justify the method
  - A. whole numbers

(Grades 6-8)

PO 1. Design a method with a series of defined steps for solving a problem; justify the method
   B. fractions, decimals and integers

- **6M-E3. Use *if . . . then* statements to construct simple valid arguments**

(Grades 4-5)

PO 1. Construct simple valid arguments using *if* . . . *then* statements based on
   A. graphic organizers (e.g., Venn diagrams and pictures . . .)
   B. geometric shapes

(Grades 6-8)

PO 1. Construct simple valid arguments using *if* . . . *then* statements based on
   B. geometric shapes
   C. proportional reasoning in probability
   D. syllogism

PO 2. Solve problems using deductive reasoning

MATHEMATICS GLOSSARY

Absolute Value A number's distance from zero on a number line. The absolute value of –4 is 4; the absolute value of 4 is 4.

Algebraic Methods The use of symbols to represent quantities and signs to represent their relationships.

Algebraic Sentence A general term for equations and inequalities.

Algorithms A mechanical procedure for performing a given calculation or solving a problem through step-by step procedures such as those used in long division.

Angle Measure The measure of the space between two lines that meet in a point. Angles are measured in degrees or radians.

Axiomatic Systems Systems that include self-evident truths; truths without proof and from which further statements, or theorems, can be derived.

Binomial In algebra, an expression consisting of two terms connected by a plus or minus sign, such as $4a + 6$.

Box-and-Whisker Plot A graphic method for showing a summary of data using median, quartiles and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

Census The count of a population.

Combinations Subsets of a larger number of items (e.g., the number of different teams of three that can be chosen from a group of 21).

Computational Techniques Operations or tools–number lines, calculators.

Complex Numbers Numbers that have the form $a + bi$ where a and b are real numbers and i is an imaginary number.

Congruence The state of having the same size and shape.

Conjecture An inference drawn from observed patterns in several examples.

Contextual Situation Relating mathematical problems to real, modeled or illustrated circumstance.

Coordinate System Any set of two or more magnitudes used to locate points, lines or curves. Commonly placed by using a horizontal axis (x-axis) and vertical axis (y-axis).

Correlation Coefficient A statistical measure that relates how well a set of data points can be modeled by a line.

Cosine The trigonometric function that is defined as the ratio of the leg adjacent to an angle to the hypotenuse of its right triangle.

Counterexample An example of a conditional statement in which the hypothesis is true and the conclusion is false.

Curve Fitting Plotting data and observing the pattern to predict trends.

Deductive Reasoning A series of logical steps in which a conclusion is drawn directly from a set of statements that are known or assumed to be true.

Dilation A transformation that either enlarges or reduces a geometric figure proportionately.

Direct Proof A conclusion proved through deductive reasoning.

Discrete Math The study of mathematical properties of sets and systems that have only a specific number of elements. For example, the results of tossing dice form a discrete set of events, since a die has to land on one of its six faces.

Empirical Relating to the collection of actual data.

Equation A mathematical statement in which one expression is equal to another.

Euclidean Transformations In geometry, the process of changing one configuration into another, including slides, rotations and reflections.

Exponent Tells how many times a number or variable is used as a factor. For example, 6 with an exponent of 3 (6^3) indicates that 6 is a factor 3 times (6 x 6 x 6).

Exponential Function A function commonly used to study growth and decay. It has a form $y = a^x$.

Expression A mathematical phrase with no equal sign, such as $3x$, 6, $2n + 3m$.

Factors Any of two or more quantities that are multiplied together.

Finite Graph A structure consisting of vertices and edges, where the edges indicate a mapping among the vertices (e.g., the vertices may represent players in a tournament, and the edges indicate who plays whom).

Flip A transformation, also called a reflection, that produces a mirror image of a geometric figure.

Fractal An algebraically generated complex geometric shape having the property of being endlessly self-similar under magnification. Some computer screen savers utilize fractals.

Function A dependent relationship between two sets of numbers in which a value in the first set has only one defined element in the second set.

Identify To state, match, select, write.

Imaginary Numbers The square root of a negative number usually expressed using *i*, e.g., $(\sqrt{-9}) = 3i$.

Indirect Proof A deductive proof using contradiction or elimination to rule out all except the desired conclusion.

Inductive Reasoning A form of reasoning from individual cases to general ones or from observed instances to unobserved ones.

Inequalities Statements indicating that two quantities are not equal, utilizing symbols > (greater than) or < (less than) and $\neq$.

Integers A set of numbers consisting of the whole numbers and their opposites {. . . –2, -1, 0, 1, 2 . . .}.

Inverse A related but opposite process or number such as multiplication being the inverse of division and 2/1 being the inverse of 1/2.

Irrational Numbers A set of numbers that cannot be represented as an exact ratio of two integers. For example, the square root of 2.

Iterative Processes In discrete math, a method of calculating an amount by using an initial value and applying a function repeatedly.

Linear Function A function that has a constant rate of change and can be modeled by a straight line.

Logarithm An alternative notation for expressing an exponent.

Logic A system of reasoning used to validate arguments.

Magnitude Size or quantity.

Manipulatives A wide variety of physical materials and supplies that students use to foster the learning of abstract ideas in mathematics.

Matrices A rectangular array of numbers or letters arranged in rows and columns.

Mean In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

Measures of Central Tendency Numbers that communicate the "center" or "middle" of a set of data. The mean, median and mode are statistical measures of central tendency.

Median In statistics, the quantity designating the middle value in a set of numbers.

Mode In statistics, the value that occurs most frequently in a given series of numbers.

Model (noun) A display of concrete materials, objects or drawings.

Model (verb) Use of concrete materials, symbolic.

Monomial In algebra, an expression consisting of a single term such as $5y$.

Multiple A number into which another number may be divided with no remainder.

Nonstandard Measurement Measurement expressed in terms of objects such as paper clips, sticks of gun, shoes, etc.

Normal Curve In statistics, the distribution of data along a bell-shaped curve that reaches its maximum height at the mean.

Open Sentence A statement that contains at least one unknown. For example, $6 + x = 14$.

Parallelism The state of being parallel, not intersecting.

Parameter A quantity whose value varies with the circumstances of its application, such as the radius of a group of circles.

Permutations Ordered arrangements of a given number of items in a set.

Polynomial In algebra, an expression consisting of two or more terms such as $x^2 - 2xy + y^2$.

Primes Counting numbers that can only be evenly divided by two numbers which are the number itself and 1. For example, the numbers 2, 3, 5, 7.

Proportion An equality between ratios. For example, $2/6 = 3/9$.

Quadratic Function A function that has an equation of the form $y = Ax^2 + Bx + C$ where A does not equal 0.

Radian The size of the central angle of a circle when the arc length equals the radius.

Random Variable A quantity that can take any one of a number of unpredicted values.

Range In statistics, the difference between the greatest and smallest values in a set of data.

Ratio A comparison expressed as indicated division. For example, there is a ratio of three boys to two girls in our class (3/2, 3:2).

Rational Numbers Numbers that can be expressed as an exact ratio of two integers.

Real Numbers All rational and irrational numbers.

Rectangular Array An organized arrangement of square units (tiles).

Recurrence Relations In discrete mathematics, a value in a series is derived by applying a formula to the previous value.

Recursive Sequence In discrete mathematics, a series of numbers in which values are derived by applying a formula to the previous value.

Reflection In geometry, a transformation, also called a flip, that produces a mirror image of a geometric figure.

Rotation In geometry, a transformation that turns a figure about a point.

Sample A part of the total population. Used in statistics to make predictions about the characteristics of the entire group.

Scatter Plots A graph of the points representing a collection of data.

Scientific Notation A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10.

Similarity In geometry, objects or figures that are the same shape but not necessarily the same size.

Sine A trigonometric function that is defined as the ratio of the leg opposite the angle to the hypotenuse of its right triangle.

Skip Counting Counting by equal intervals.

Slides In geometry, a transformation where a figure moves in a given direction.

Square Root Two equal factors of a number. For example, 4 is the square root of 16.

Standard Deviation A statistic that measures the dispersion of a sample.

Stem-and-Leaf Plot A table utilizing digit(s) of a number as stems and the other digit(s) as leaves. For example, 5 | 7, 8 shows 57 and 58.

Survey Interview, questionnaire and/or polling.

Symmetry A correspondence in size, form and arrangement of parts on opposite sides of a plane, line or point. For example, a figure that has line symmetry has two halves that coincide if folded along its line of symmetry.

Synthetic Representation The geometric form as opposed to the algebraic representation of a figure.

Tangent A trigonometric function of an angle which is defined as the ratio of the lengths of the leg opposite to the leg adjacent to an angle in its right triangle. Also a line having one point in common with a curve.

Tessellations A mosaic formed by repetitions of a single shape.

Theoretical (mathematical) Relating to the probability of a given event, using mathematical relationships (e.g., the chance of a red side coming up on the flip of a two-colored counter is one in two or 1/2).

Transformation A geometric process for changing one figure into another.

Trigonometric Ratios The ratios of the lengths of pairs of sides in a right triangle, i.e., sine, cosine and tangent.

Trigonometry The branch of mathematics involving triangles that combines arithmetic, algebra and geometry. Trigonometry is used in surveying, navigation and physics.

Validity An argument that is correctly inferred or deduced from a premise.

Variability Numbers that describe how spread out a set of data is (e.g., range and quartile).

Variable A place holder in algebraic expressions. In $3x + y = 23$, x and y are variables.

Vector Quantity that has magnitude (length) and direction. It may be represented as a directed line segment (→).

Whole Numbers The counting numbers and zero {0, 1, 2, 3 . . .}.